

Rye en Reekse : Deel 1

Rekenkundig: $d = T_2 - T_1 = T_3 - T_2$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

Meetkundig: $r = \frac{T_2}{T_1} = \frac{T_3}{T_2}$

$$T_n = a \cdot r^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} / S_n = \frac{a(1 - r^n)}{1 - r}$$

1) Gegee: 5 ; 8 ; 11

$$\begin{array}{ccc} \underbrace{5} & & \underbrace{8} \\ & +3 & +3 = d \end{array}$$

1.1) Bepaal die waarde van die 10de term.

1.2) Bepaal die som van die eerste 10 terme.

$$1.1) T_n = a + (n-1)d$$

$$T_{10} = 5 + (10-1)(3)$$

$$T_{10} = 5 + 9(3)$$

$$T_{10} = 32$$

$$1.2) S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{10} = \frac{10}{2} [2(5) + (10-1)(3)]$$

$$S_{10} = 5 (10 + 9(3))$$

$$S_{10} = 185$$

2) Gegee: 4 ; -12 ; 36

$$x-3 \quad x-3 = r$$

2.1) Watter term sal 'n waarde van 26244 hê?

2.2) Hoeveel terme moet opgetel word om 'n som van 2188 te gee?

$$\begin{aligned} 2.1) \quad T_n &= a \cdot r^{n-1} \\ 26244 &= (4)(-3)^{n-1} \\ (6561) &= (-3)^{n-1} \\ 3^8 &= (-3)^{n-1} \\ 8 &= n-1 \end{aligned}$$

$$n = 9$$

$$T_9 = 26244$$

$$2.2) \quad S_n = \frac{a(r^n - 1)}{r - 1}$$

$$2188 = \frac{4((-3)^n - 1)}{-3 - 1} = \frac{\cancel{4}((-3)^n - 1)}{-\cancel{4}}$$

$$2188 = -(-3)^n + 1$$

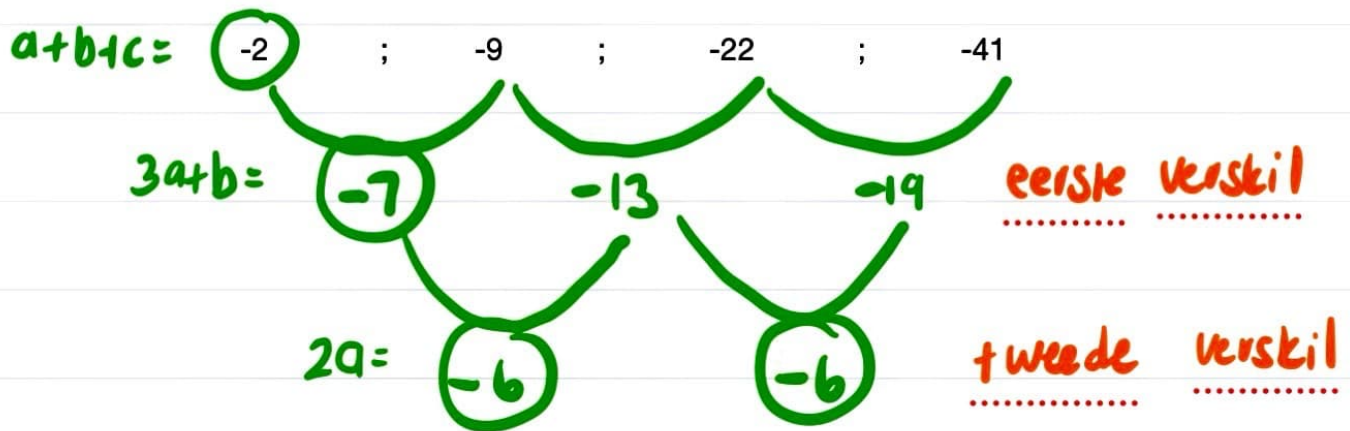
$$2187 = -(-3)^n$$

$$-2187 = (-3)^n$$

$$-3^7 = (-3)^n$$

$$7 = n$$

3) Bepaal 'n formule vir die n-de term van:



$$T_n = an^2 + bn + c$$

$$2a = -6$$
$$\therefore a = -3$$

$$3a + b = -7$$
$$3(-3) + b = -7$$
$$b = 2$$

$$a + b + c = -2$$
$$-3 + 2 + c = -2$$
$$c = -1$$

$$\therefore T_n = -3n^2 + 2n - 1$$

T_1 T_2 T_3
 4) Indien x ; $4x + 5$; $10x - 5$ die eerste drie terme van 'n rekenkundige ry is, bepaal die waarde van x .

$$d = T_2 - T_1 = T_3 - T_2$$

$$4x + 5 - x = 10x - 5 - (4x + 5)$$

$$3x + 5 = 10x - 5 - 4x - 5$$

$$3x + 5 = 6x - 10$$

$$-3x = -15$$

$$x = 5$$

T_1 T_2 T_3
 5) Indien $k + 1$; $k - 1$; $2k - 5$ die eerste drie terme is van 'n meetkundige ry is, bepaal die waarde van k .

$$r = \frac{T_2}{T_1} = \frac{T_3}{T_2}$$

$$\frac{k-1}{k+1} = \frac{2k-5}{k-1}$$

$$(k-1)(k-1) = (2k-5)(k+1)$$

$$k^2 - 2k + 1 = 2k^2 - 3k - 5$$

$$0 = k^2 - k - 6$$

$$0 = (k-3)(k+2)$$

$$\therefore k = 3 \text{ of } k = -2$$

6) Bepaal die waarde van x en y indien die volgende patroon 'n kwadratiese ry is:

-2 ; x ; 8 ; 19 ; y

$x+2$ $8-x$ 11 $y-19$

$8-x-(x+2) = 11-(8-x) = y-19-11$ *+ tweede verskil*

$$\begin{aligned}8-x-(x+2) &= 11-(8-x) \\8-x-x-2 &= 11-8+x \\6-2x &= 3+x \\3 &= 3x \\\therefore x &= 1\end{aligned}$$

$$\begin{aligned}11-(8-(1)) &= y-19-11 \\11-7 &= y-30 \\4 &= y-30 \\\therefore 34 &= y\end{aligned}$$

KONVERGEER

Indien $-1 < r < 1$

$S_{\infty} = \frac{a}{1-r}$

Gegee: $(2t - \frac{2}{3})^2 + (2t - \frac{2}{3})^3 + (2t - \frac{2}{3})^4 + \dots$

A) Bepaal die waardes van t waarvoor hierdie reeks sal konvergeer.

B) Indien $t = 1/2$, bepaal die som tot oneindig.

1)

$-1 < r < 1$

$$r = \frac{T_2}{T_1} = \frac{(2t - \frac{2}{3})^3}{(2t - \frac{2}{3})^2} = 2t - \frac{2}{3}$$

$$\begin{aligned}
 -1 &< 2t - \frac{2}{3} < 1 \\
 -\frac{1}{3} &< 2t < \frac{5}{3} \\
 -\frac{1}{6} &< t < \frac{5}{6}
 \end{aligned}$$

$$\begin{aligned}
 B) \quad S_{\infty} &= \frac{a}{1-r} \\
 S_{\infty} &= \frac{\frac{1}{9}}{1 - \frac{1}{3}}
 \end{aligned}$$

$$S_{\infty} = \frac{1}{6}$$

$$\begin{aligned}
 a &= (2t - \frac{2}{3})^2 \\
 a &= (2(\frac{1}{2}) - \frac{2}{3})^2 = \frac{1}{9}
 \end{aligned}$$

$$\begin{aligned}
 r &= 2t - \frac{2}{3} \\
 r &= 2(\frac{1}{2}) - \frac{2}{3} = \frac{1}{3}
 \end{aligned}$$

SIGMA Σ

$$S_n$$

Bv.

$$\sum_{k=3}^6 (2k-1) = T_n$$

✓ (+) eerste 3 terme

✓ (*) $n = \text{bo-onder} + 1$

$$\begin{array}{ccc} T_1 & T_2 & T_3 \\ 5 & 7 & 9 \\ & +2 & +2 = d \end{array}$$

$$S_n = \frac{n}{2} [2a + (n-1)d] \quad n = 6 - 3 + 1 = 4$$

$$S_4 = \frac{4}{2} [2(5) + (4-1)(2)] = 32$$

1) $\sum_{k=0}^7 (3 \cdot 2^k)$

$$\begin{array}{ccc} T_1 & T_2 & T_3 \\ 3 & 6 & 12 \\ & \times 2 & \times 2 = r \end{array}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

* $n = 7 - 0 + 1 = 8$

$$S_8 = \frac{3((2)^8 - 1)}{2 - 1} = 765$$

2) $\sum_{k=2}^m \left(\frac{1}{15} \cdot 3^{k-1} \right) = 24 \frac{1}{5}$ Bepaal die waarde van m .

T_1 T_2 T_3
 $\left(\frac{1}{5} \right)$ $\frac{3}{5}$ $\frac{9}{5}$
 $\times 3$ $\times 3 = r$

$S_n = \frac{a(r^n - 1)}{r - 1}$
 $24 \frac{1}{5} = \frac{\frac{1}{5} (3^m - 1)}{3 - 1}$
 $\left(\frac{121}{5} \right) (2) + 1 = 3^{m-1}$

$n = m - 2 + 1 = m - 1$

$243 = 3^{m-1}$
 $3^5 = 3^{m-1}$
 $5 = m - 1$
 $\therefore m = 6$

3) Skryf die volgende reeks in sigma notasie:

$\sum_{n=1}^{20} (4n - 1)$

$3 + 7 + 11 + \dots + 79$
 $+4 \quad +4 = d$

$T_n = a + (n-1)d$
 $T_n = 3 + (n-1)(4)$
 $T_n = 3 + 4n - 4$
 $T_n = 4n - 1$

$79 = 4n - 1$

$80 = 4n$

$n = 20$

4) $\sum_{k=1}^{\infty} 10 \cdot (2)^{3-k}$

T_1 T_2 T_3
 40 20 10
 $\times \frac{1}{2}$ $\times \frac{1}{2} = r$

$$S_{\infty} = \frac{a}{1-r}$$

$\textcircled{*} \eta = \infty - 1 + 1 = \infty$

$$S_{\infty} = \frac{40}{1 - \frac{1}{2}} = 80$$



As $S_n = 3n(n-4)$ vind die waarde van die 10de term.

T_{10}

$$T_{10} = S_{10} - S_9$$



$$S_{10} = 3(10)(10-4) = 180$$

$$S_9 = 3(9)(9-4) = 135$$

$$T_{10} = 180 - 135$$

$$T_{10} = 45$$