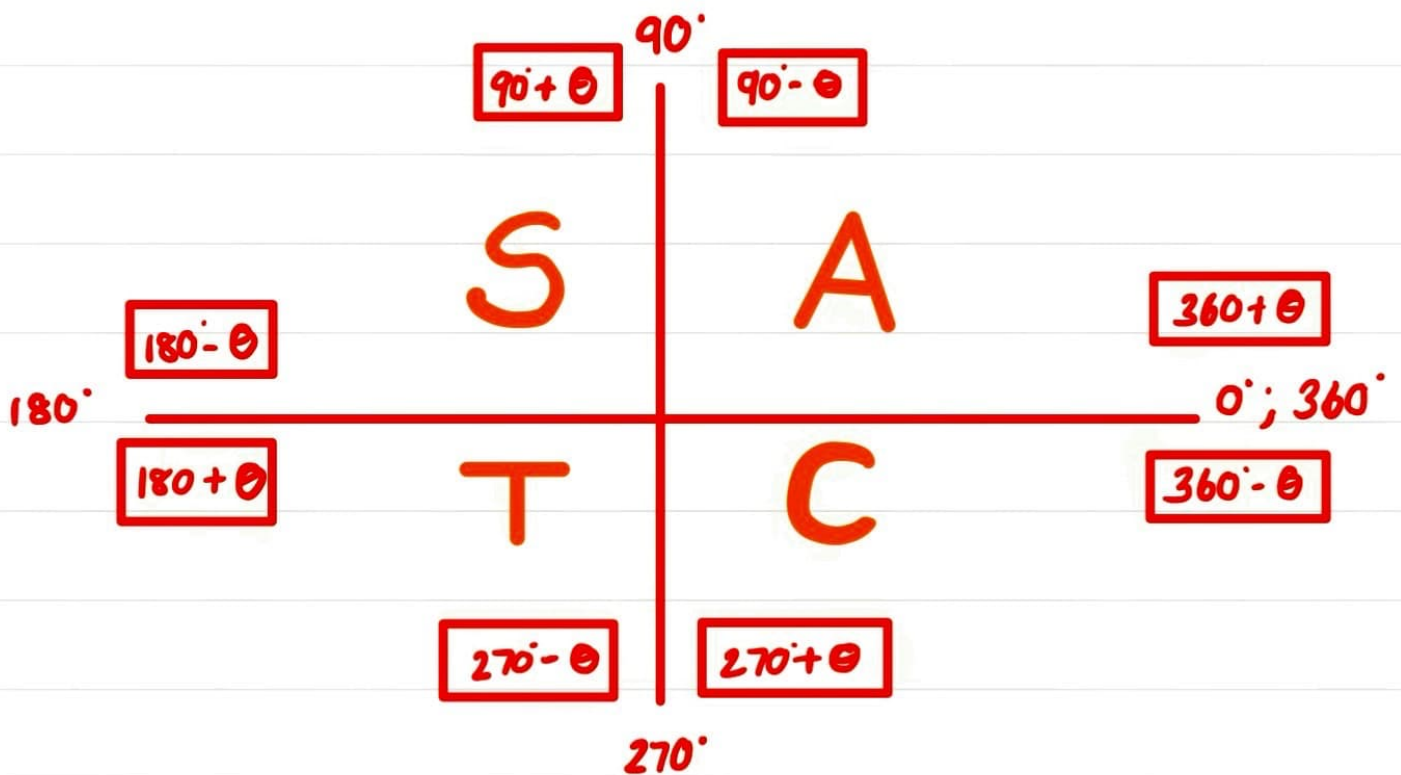
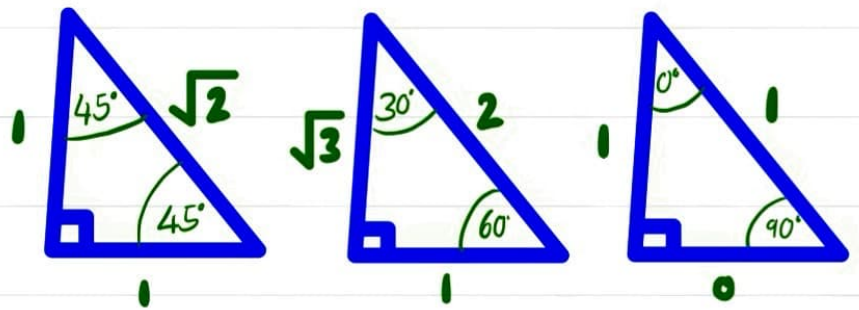


# Trigonometrie Deel 1 : Algemene beginsels

| sin | cos | tan |
|-----|-----|-----|
| T   | A   | T   |
| S   | S   | A   |



Identiteite:

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\rightarrow \sin^2 \theta = 1 - \cos^2 \theta$$

$$\rightarrow \cos^2 \theta = 1 - \sin^2 \theta$$

## Gr. 12 saamgestelde hoek identiteite

$$\bullet \sin(A+B) = \sin A \cos B + \cos A \sin B$$

\*

$$\bullet \sin(A-B) = \sin A \cos B - \cos A \sin B$$

\*

$$\bullet \cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\bullet \cos(A-B) = \cos A \cos B + \sin A \sin B$$

Vereenvoudig sonder die gebruik van 'n sakrekenaar:

$$\textcircled{1} \quad \sin(\overset{A}{x} + \overset{B}{60}) - \sin(\overset{A}{x} - \overset{B}{60})$$

$$= \sin x \cos 60 + \cos x \sin 60 - (\sin x \cos 60 - \cos x \sin 60)$$

$$= \cancel{\sin x \cos 60} + \cos x \sin 60 - \cancel{\sin x \cos 60} + \cos x \sin 60$$

$$= 2 \cos x \sin 60$$

$$= 2 \cos x \left( \frac{\sqrt{3}}{2} \right)$$

$$= \sqrt{3} \cos x$$

②  $\sin 50^\circ \cdot \sin 20^\circ + \cos 20^\circ \cdot \cos 50^\circ$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\begin{aligned} &= \cos 20^\circ \cdot \cos 50^\circ + \sin 50^\circ \cdot \sin 20^\circ \\ &= \cos 50^\circ \cdot \cos 20^\circ + \sin 50^\circ \cdot \sin 20^\circ \\ &= \cos(50^\circ - 20^\circ) \\ &= \cos 30^\circ \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

③  $\cos 40^\circ \cdot \sin 10^\circ - \sin 40^\circ \cdot \cos 10^\circ$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\begin{aligned} &= \sin 10^\circ \cdot \cos 40^\circ - \cos 10^\circ \cdot \sin 40^\circ \\ &= \sin(10^\circ - 40^\circ) \\ &= \sin(-30^\circ) \\ &= \sin(360^\circ - 30^\circ) \\ &= -\sin 30^\circ \\ &= -\frac{1}{2} \end{aligned}$$



$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

④

$$-\cos 15^\circ \sin 30^\circ - \cos 75^\circ \cos 30^\circ$$

$$\begin{aligned} &= -(\cos 75^\circ \cos 30^\circ + \cos 15^\circ \sin 30^\circ) \\ &= -(\sin 15^\circ \cos 30^\circ + \cos 15^\circ \sin 30^\circ) \\ &= -(\sin(15+30)) \\ &= -\sin 45^\circ \\ &= -\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2} \end{aligned}$$

30° ; 45° ; 60° ; 90°

⑤.1

$\sin 75^\circ$

$$= \sin(45^\circ + 30^\circ)$$

$$\begin{aligned} &= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ \\ &= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) \\ &= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} \\ &= \frac{(\sqrt{3} + 1) \times \sqrt{2}}{2\sqrt{2} \times \sqrt{2}} \\ &= \frac{\sqrt{6} + \sqrt{2}}{4} \end{aligned}$$

⑤.2

$\sin 15^\circ$

$$= \sin(45^\circ - 30^\circ)$$

$$\begin{aligned} &= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ \\ &= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) \\ &= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} \\ &= \frac{(\sqrt{3} - 1) \times \sqrt{2}}{2\sqrt{2} \times \sqrt{2}} \\ &= \frac{\sqrt{6} - \sqrt{2}}{4} \end{aligned}$$

$$6) \frac{\sqrt{3} \cos B + \sin B}{2}$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$= 2 \left( \frac{\sqrt{3}}{2} \cos B + \frac{1}{2} \sin B \right)$$

$$= 2 \left( \overset{A}{\cos 30^\circ} \overset{B}{\cos B} + \overset{A}{\sin 30^\circ} \overset{B}{\sin B} \right)$$

$$= 2 \cos(30^\circ - B)$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$7) \sin 50^\circ + \cos 50^\circ \tan 20^\circ$$

$$= \sin 50^\circ + \frac{\cos 50^\circ \sin 20^\circ}{\cos 20^\circ}$$

$$= \frac{\overset{A}{\sin 50^\circ} \overset{B}{\cos 20^\circ} + \overset{A}{\cos 50^\circ} \overset{B}{\sin 20^\circ}}{\cos 20^\circ}$$

$$= \frac{\sin(50^\circ + 20^\circ)}{\cos 20^\circ}$$

$$= \frac{\sin 70^\circ}{\cos 20^\circ}$$

$$= \frac{\cancel{\cos 20^\circ}}{\cancel{\cos 20^\circ}} = 1$$

## Gr.12 Dubbelhoek identiteite

$$\bullet \sin 2A = 2 \sin A \cdot \cos A \bullet$$

\*

$$\begin{aligned} \bullet \cos 2A &= 1 - 2\sin^2 A \\ &= 2\cos^2 A - 1 \\ &= \cos^2 A - \sin^2 A \bullet \end{aligned}$$

\*

Vereenvoudig sonder die gebruik van 'n sakrekenaar:

$$\begin{aligned} \textcircled{1} \quad & \frac{\cos 2B + 1}{2\cos B} \\ &= \frac{\cancel{2\cos^2 B} - \cancel{1} + \cancel{1}}{\cancel{2\cos B}} \\ &= \cos B \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad & \cos^2 15^\circ - \sin^2 15^\circ \\ &= \cos 2(15^\circ) \\ &= \cos 30^\circ \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad & \frac{\sin 42^\circ}{\sin 14^\circ} - \frac{\cos 42^\circ}{\cos 14^\circ} \\ &= \frac{2(\sin^{A} 42^\circ \cdot \cos^{B} 14^\circ - \cos^{A} 42^\circ \cdot \sin^{B} 14^\circ)}{2 \sin 14^\circ \cdot \cos 14^\circ} \\ &= \frac{2 \sin(42^\circ - 14^\circ)}{\sin 2(14^\circ)} = \frac{2 \sin 28^\circ}{\sin 28^\circ} \\ &= 2 \end{aligned}$$

$$\begin{aligned} \textcircled{4} \quad & -\cos^2 15^\circ - \sin^2 15^\circ \\ &= -(\cos^2 15^\circ + \sin^2 15^\circ) \\ &= -1 \end{aligned}$$



$$* \quad 2\cos^2 A - 1 = \cos 2A$$

$$2\sin A \cos A = \sin 2A$$

$$⑤ \quad 1 - 2\cos^2 \frac{x}{2}$$

$$\begin{aligned} &= - (2\cos^2 \frac{x}{2} - 1) \\ &= - \cos 2 \left( \frac{x}{2} \right) \\ &= - \cos x \end{aligned}$$

$$⑥ \quad \frac{\sin 15^\circ \cdot \cos 15^\circ}{2}$$

$$\begin{aligned} &= \frac{\sin 2(15^\circ)}{2} = \frac{\sin 30^\circ}{2} \\ &= \frac{\frac{1}{2}}{2} = \frac{1}{4} \end{aligned}$$

$$\cos 2A = 1 - 2\sin^2 22,5^\circ$$

$$\begin{aligned} ⑦ \quad &2\cos^2 15^\circ - 1 + 1 \\ &\cos 2(15^\circ) + 1 \\ &= \cos 30^\circ + 1 \\ &= \frac{\sqrt{3}}{2} + 1 \\ &= \frac{\sqrt{3} + 2}{2} \end{aligned}$$

$$\begin{aligned} ⑧ \quad &\frac{1}{2} - \sin^2 22,5^\circ \\ &= \frac{1 - 2\sin^2 22,5^\circ}{2} \\ &= \frac{\cos 2(22,5^\circ)}{2} = \frac{\cos 45^\circ}{2} \\ &= \frac{1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4} \end{aligned}$$

$$⑨ \quad (\sin 22,5^\circ - \cos 22,5^\circ)(\sin 22,5^\circ + \cos 22,5^\circ)$$

$$= \sin^2 22,5^\circ + \cancel{\sin 22,5^\circ \cdot \cos 22,5^\circ} - \cancel{\sin 22,5^\circ \cdot \cos 22,5^\circ} - \cos^2 22,5^\circ$$

$$= \cos 2A = \cos^2 A - \sin^2 A$$

$$\begin{aligned} &= - (\cos^2 22,5^\circ - \sin^2 22,5^\circ) \\ &= - \cos 2(22,5^\circ) = - \cos 45^\circ \\ &= - \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = - \frac{\sqrt{2}}{2} \end{aligned}$$